

FREQUENCY-DEPENDENT ATTENUATION OF SOILS IN SEISMIC SIMULATION

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ABSTRACT

A seismic simulation is performed on a synthetic soil-rock model with frequency-dependent attenuation spectra of soils. A detailed attenuation (i.e., quality factor, Q) characterization of remolded kaolin is obtained by utilizing a newly developed non-resonance experimental method. The attenuation spectra exhibit an apparent and consistent hump-shape frequency-dependent pattern in the seismic frequency range and demonstrate general and quantitative similarity to previously studied natural soils of different geologic origins. This frequency-dependent attenuation behavior cannot be described with the simplified linear relationships with respect to frequency for modeling applications. A frequency-dependent attenuation spectrum of soils is established in a polynomial formula in the frequency range of 10^{-2} to 30 Hz. To quantify the frequency-dependent effect on the amplification, a transfer function which is established for vertically propagating SH waves passing through a single soil layer underlain by bedrock is used. The results from the simulation indicate that the accurate characterization of frequency-dependent attenuation is instrumental in amplification spectra estimate; whereas constant Q assumption may overestimate the amplification spectra at a maximum of 7 times at 30 Hz for SH waves traveling through a soil layer of 100 m thick.

Introduction

Less consolidated soils (e.g., sands, silts, and clays) respond to seismic waves much differently than bedrock and often change the amplitude and frequency content of seismic ground motions. The degree of ground motion amplification depends on the soil type, properties and thickness. It also depends on the match between the frequency content of the earthquake motions and the natural period(s) of the ground (i.e., resonance). Amongst the fundamental contributing factors of the site amplification, lower impedance of soil layers appears to be dominant, whilst higher intrinsic attenuation incurs seismic energy dissipation and restrains the amplification amplitude. Thus, an accurate characterization of the seismic dispersion and

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attenuation of soils is essential to a better understanding of seismic topics.

Efforts have been maintained for decades in studying the seismic behavior of soils and other less consolidated geological materials and an extensive literature exists. It has been concluded or reinforced that the seismic anelastic attenuation, Q , of dry and saturated sands, is frequency-independent or a linear function of frequency, with a little measurable dispersion (Hardin and Drnevich, 1972; Bolton and Wilson, 1989). However, perspectives based on measurements with the aid of interpolation regarding frequency-dependency split silts from clays in the seismic frequency range from several Hertz to dozens of Hertz (Hamilton, 1976; Stoll, 1985; Stokoe, 1999). On the other hand, some independent evidence collected by Anderson and Hough (1984) support the constant Q regardless of components of the geological materials under discussion. This debate is mostly due to lack of direct measurements of seismic behavior in the seismic frequency range under consideration. Common practices to overcome the sparseness of measurements include: excessive interpolation based on one to two measurements per frequency decade; extrapolation based on quantities obtained in sonic frequency ranges; or a combination power of measurements from different test methods. Many experimental techniques such as resonance method and cyclic loading method and field methods have been continuously improved (Woods, 1994) and appear to provide achievable solutions. However, most of these methods are single-point or narrow band measurements which are limited by their testing mechanisms and are thus not likely equipped with spectral measurement capacity in the seismic frequency range under consideration. This situation is echoed by Kibblewhite (1989), who considers it “fraught with traps and difficulties”. This paper reviews some new experimental results obtained from a recently developed non-resonance method between 10^{-2} and 30 Hz with a group of four remolded kaolin specimens. The results illustrate that both the shear wave velocity, V_s , and the quality factor, Q , are frequency-dependent. Afterward, characteristic spectra are extracted by performing regression from the measurements. Finally, we conduct a seismic site amplification comparison by using the frequency-dependent data and a constant Q dataset with a synthetic single soil layer model.

Frequency-Dependent Observation from Non-Resonance Tests

Non-resonance method was theoretically considered the best alternative in studying the seismic behavior of geological materials in the frequency range of seismic interest. However, accompanying technical practice was reportedly very difficult (Kibblewhite, 1989). The main concept of this method was originally established in mechanical engineering studies of polymers and biomaterials, which typically exhibit significant linear viscoelasticity in common testing conditions. The ratio between loading stress and resulting strain is used to deduce a complex elastic modulus, from which velocity and quality factor can be subsequently extracted. For a forced harmonic torsional oscillation of a viscoelastic cylinder at small amplitudes as shown in Fig. 1, a transfer function between the excitation, $T_0 e^{i\omega t}$, and response, $\phi(0, t)$, of a specimen is obtained by:

$$\frac{T_0 e^{i\omega t}}{\phi(0, t)} = \frac{\pi R^4}{2} \rho \omega^2 l \frac{\cot \Omega^*}{\Omega^*} - J_0 \omega^2 \quad (1)$$

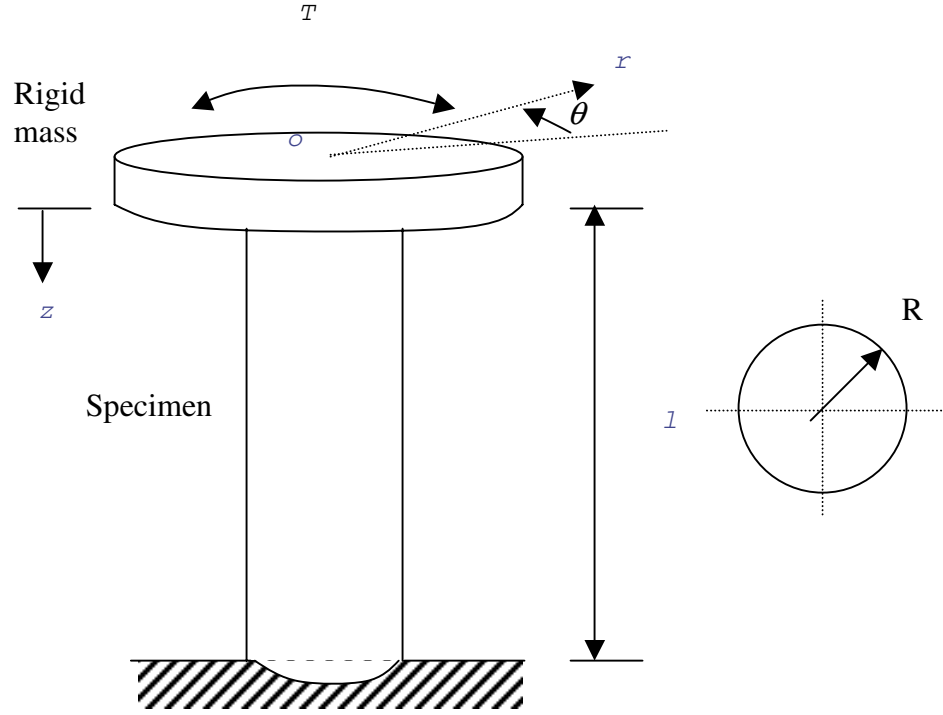


Figure 1. Fixed-Free Boundary Model Representing the Non-Resonance Method

where $\Omega^*(\omega) = \sqrt{\frac{\rho \omega^2 l^2}{G^*(\omega)}}$, ω is circular frequency, R , l , ρ and J_0 are radius, length, density of the test specimen, and the mass polar moment of the electromagnetic motor, respectively. By definition, the complex shear modulus is $G^*(\omega) = G_1(\omega) + i \cdot G_2(\omega)$, where $G_1(\omega)$ and $G_2(\omega)$ are storage and loss modulus, respectively. The shear wave velocity $V_s(\omega)$ and the quality factor $Q(\omega)$ can be calculated using the following definitions, respectively:

$$V_s^*(\omega) = \sqrt{\frac{G^*(\omega)}{\rho}} \quad (2a)$$

$$Q^{-1}(\omega) = \frac{G_2(\omega)}{G_1(\omega)} \quad (2b)$$

Amongst its advantages, the non-resonance method can be configured to yield spectral measurements in the seismic frequency range of 10^{-2} to 30 Hz with reasonable effort. In addition, experiments can be performed within their linear strain amplitude of approximately 8×10^{-6} with 150 to 200 measurements per decade. An extensive calibration program was enforced during the experimentation and several technical problems have been solved successfully (for more details see Meng, 2003).

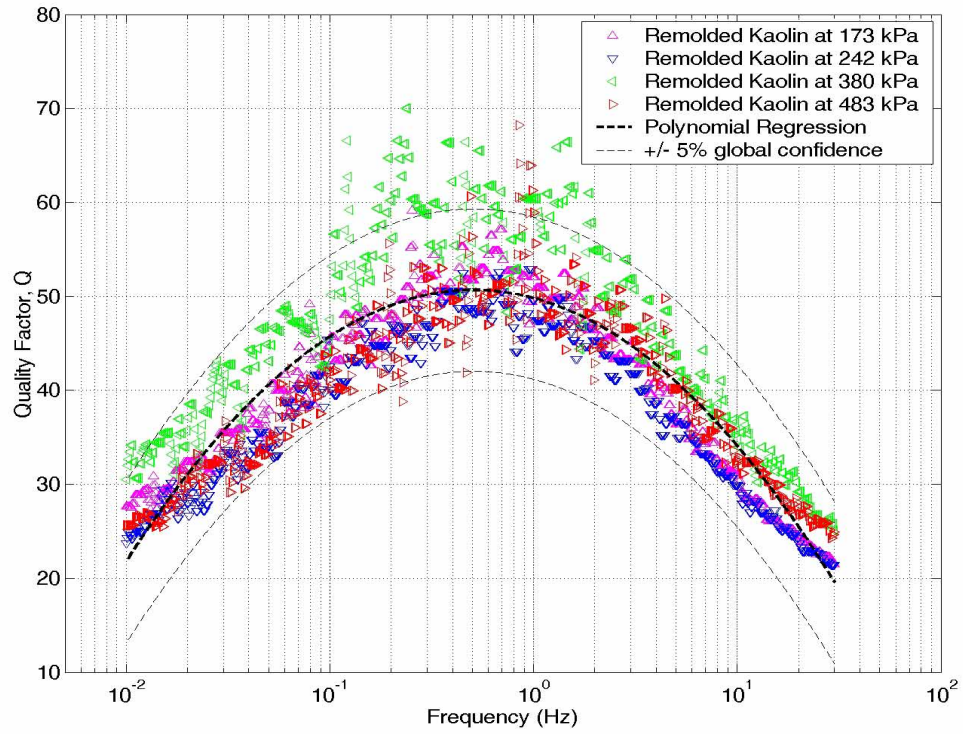


Figure 2. Quality Factor Spectra of Remolded Kaolin Specimens Tested between 10^{-2} and 30 Hz

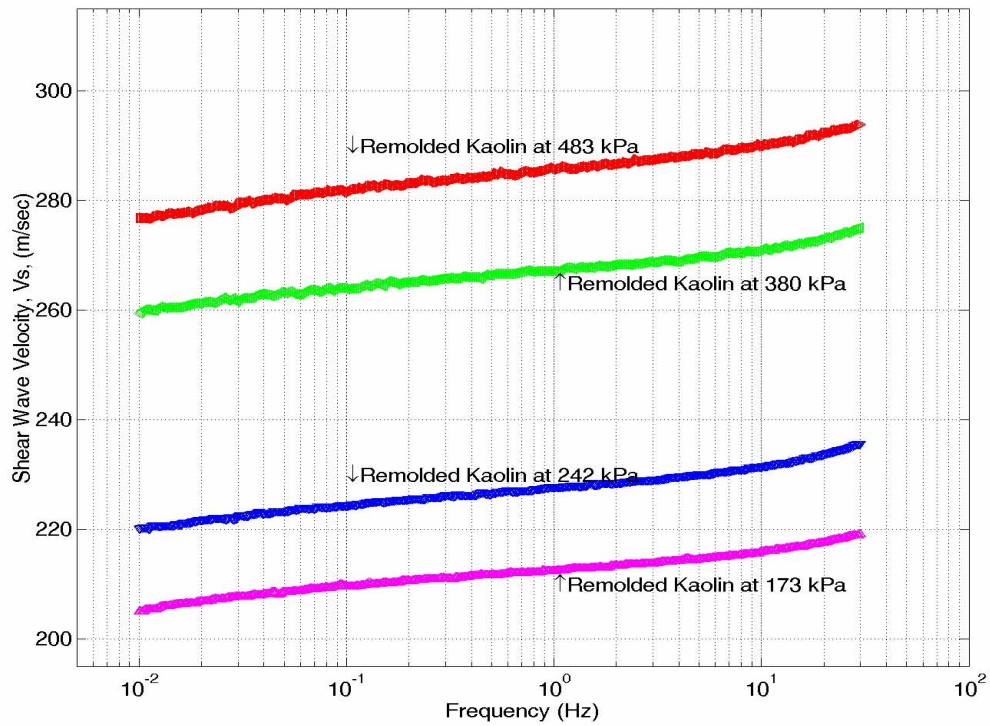


Figure 3. Shear Wave Velocity Spectra of Remolded Kaolin Specimens Tested between 10^{-2} and 30 Hz

Measurements obtained from the non-resonance method with remolded kaolin specimens are referenced in this study. Kaolin is one of the best known modeling minerals that have been broadly used in the study of static and dynamic behavior of soils. It is a mined residual soil that belongs to the Piedmont geologic region and results from weathering of feldspar-rich igneous and metamorphic rocks. The specimens are prepared at arbitrarily selected confining stresses of 173, 242, 380, and 483 kPa, which correspond to stress levels at depths of approximately 20, 30, 50, and 60 meters. The specimens have an average mass density of 1.82 Mg/m^2 and moisture content of 38%. Figs. 2 and 3 show the spectral measurements. In brevity, the four quality factor spectra have uniform hump shape with maxima around 1 Hz. Fluctuations in the quality factor spectra are mainly coming from the effect of ambient noise and is inevitably obvious regarding the strain amplitude (i.e., $< 10^{-5}$) of the measurements and the available testing environment. In spite of the effect of ambient noise, the hump-trend is still distinguishable during visual survey with the aid of dense measurements. Unlike the shear wave velocity spectra, the quality factor spectra do not appear to be sensitive to the selected preparation pressures. The four shear wave velocity spectra are monotonically increasing with respect to the frequency. The amount of increment with frequency (i.e., dispersion) is modest at an approximate rate of 2.5% per decade. It appears that the frequency-dependency in quality factor spectra is much more obvious than

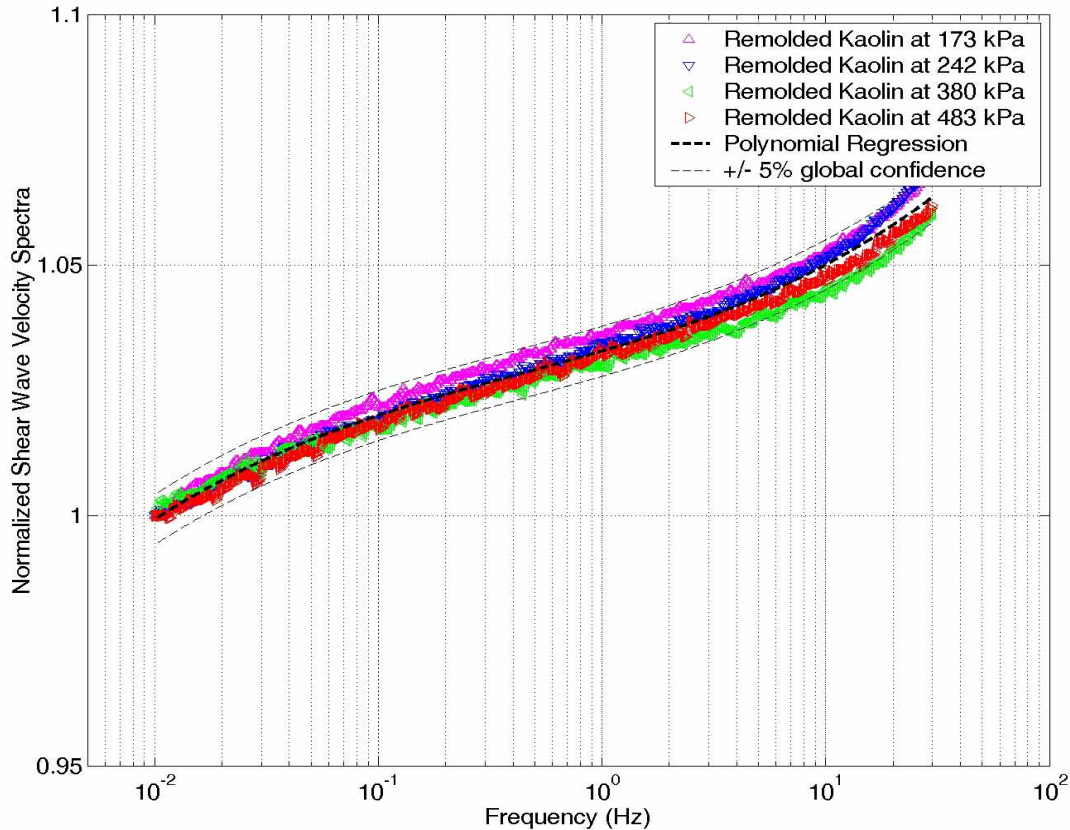


Figure 4. Normalized Shear Wave Velocity Spectra and the Characteristic Spectra Obtained from Polynomial Regression

that illustrated in the corresponding dispersion spectra. A characteristic quality factor spectrum of the remolded soils is deduced from a simple quadratic polynomial regression as shown in Fig 2. Its frequency-dependent quality factor formula can thus be described as:

$$Q(f) = -9.9 \cdot \lg^2 f - 5.8 \cdot \lg f + 49.8 \quad (3)$$

where f is frequency in Hz. A previous study (Meng and Rix, 2004) on the spectra has established that the inverse of the quality factor, attenuation coefficient, is the differential of the shear modulus, or shear wave velocity, at a frequency under consideration. The frequency dependency and pressure dependency of shear wave velocity formula can be modeled as:

$$V_s(f, \sigma) = F_1(f) \cdot F_2(\sigma) \quad (4)$$

where $F_1(f)$, $F_2(\sigma)$ and σ are the frequency-dependent component, the pressure-dependent component and confining pressure, respectively. $F_2(\sigma)$ is normalized with arbitrarily selected reference values at 10^{-2} Hz (i.e. divided by its value at 10^{-2} Hz) and the typical exponential format of other pressure-dependent formulae (Ishihara, 1996). A characteristic shear wave velocity spectrum, as shown in Fig. 4, is established using the normalized spectra. The normalized spectrum was regressed, to obtain the following $F_1(f)$ and $F_2(\sigma)$:

$$F_1(f) = 0.002 \cdot \lg^3 f + 0.002 \cdot \lg^2 f + 0.013 \cdot \lg f + 1.032$$

$$F_2(\sigma) = 42.5 \cdot \sigma^{0.3} \quad (\sigma \text{ is in kPa})$$

The Effect of Frequency-Dependent Q on Kappa, κ , Factor

High frequency amplitudes of seismic ground motion are reduced through a high-cut operator, $\exp(-\pi f \kappa)$, where kappa, κ , is a function of Q and V_s (Anderson and Hough, 1984). κ acts to rapidly diminish spectral amplitudes above some frequency, and is believed to be primarily a site effect (Atkinson, 2004). The κ factor was introduced to primarily consider near-surface (upper crust) attenuation of seismic waves, and the values of κ exhibit both a region-and a site-dependent character. For hard-rock sites in Eastern North America (ENA), κ is very low and nearly negligible; a minimum κ of zero, with a maximum value for individual records of 0.01 (Atkinson, 2004). κ factor for California varies between 0.02 and 0.04 for soft rock sites (Anderson and Hough, 1984; Boore et al, 1992; Atkinson and Silva, 1997; Boore and Joyner, 1997). Zero-distance Kappa factor is given by $\kappa_0 = \frac{H}{\bar{Q} \times \bar{V}_s}$, where $\bar{Q}$ and $\bar{V}_s$ are

averaged over a depth of H , beneath the site (Anderson and Hough, 1984). In this section we evaluate the effect of the frequency dependency of the aforementioned Q and V_s and Kappa factor and subsequently on the site amplification. Since Q and V_s are frequency dependent parameters in this study, the zero-distance for a depth of 100 meters ($H = 100$ m) is a function of frequency. We consider the average of zero-distance κ as a reference for frequency independent κ . Figure 5 compares the high-cut operator, $\exp(-\pi f \kappa_0)$, for both cases of frequency-dependent and frequency-independent cases. The increment of the high-cut operator due to the frequency dependency of κ factor is negligible below 10 Hz but becomes obvious for frequencies above 10 Hz. The observation agrees well with the theoretical interpretation of the site attenuation from soil column at high frequencies (Boore and Joyner, 1991). This effect is

not considerable for the site amplification studies, where the most interested frequency range is below 10 Hz, but can be taken into the account for cases where the site amplification is needed beyond 10 Hz.

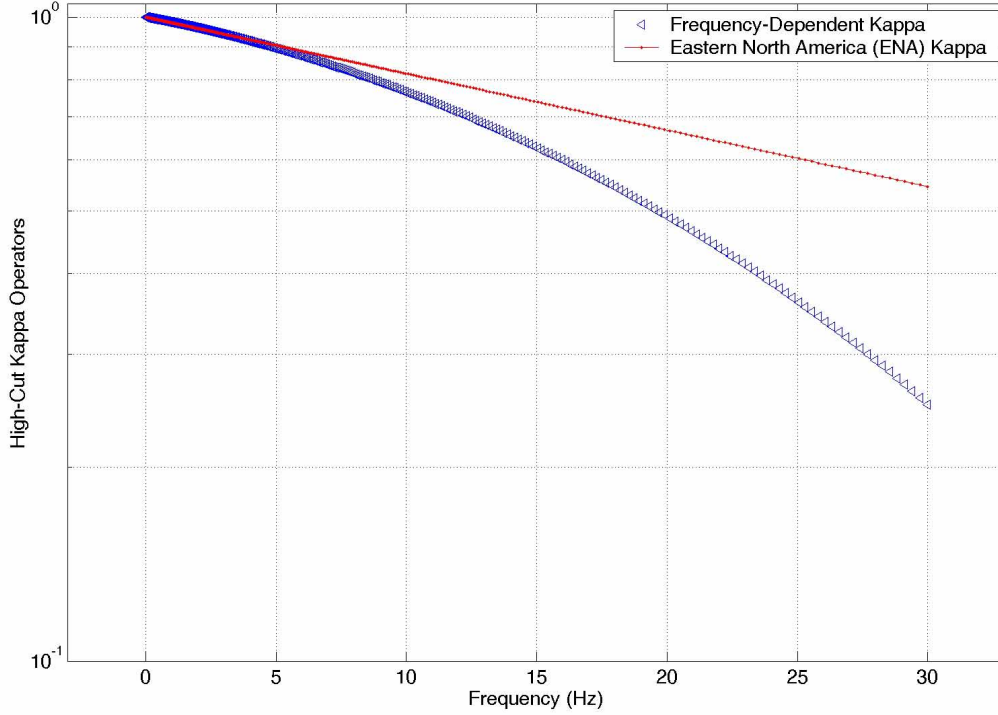


Figure 5. High-Cut Kappa Operators Comparison with Frequency-Dependent and -Independent Quality Factors

Comparison by Using A Single Soil Layer Synthetic Model

By using a synthetic model as schematically shown in Fig. 6, we compare the frequency response spectra obtained from the frequency-dependent and -independent parameters. A discrete-time transfer function as proposed by Şafak (1995), which models vertically propagating SH waves passing through a single soil layer over bedrock was selected for the comparison. The transfer function of the synthetic model can be established as:

$$H(f) = \frac{Y(f)}{X(f)} = \frac{(1 + r^*) \cdot e^{-i 2 \pi f \tau^*}}{1 + r^* \cdot e^{-i 4 \pi f \tau^*}} \quad (5)$$

where $X(f)$ and $Y(f)$ are the Fourier transforms of input $x(t)$ and output $y(t)$ time history, r^* and τ^* are complex reflection coefficient and complex traveling time, respectively. It can be shown that $r^* = \frac{4Qr - i \cdot (1-r)}{4Q + i \cdot (1-r)}$, $\tau^* = \frac{2Q}{2Q + i} \tau$, $r = \frac{\rho_r v_r - \rho_s v_s}{\rho_r v_r + \rho_s v_s}$ and $\tau = \frac{h}{v_s}$, where r , τ and h are reflection coefficient, traveling time, and the thickness respectively. ρ_s, v_s , ρ_r and v_r are the mass density and SH wave velocity in the rock and soil layer, respectively. A complex shear

wave velocity $v_s^* = v_{s1} + i \cdot v_{s2}$ was used to derive the linear viscoelastic model from a linear elastic model. In this study, the reflection coefficient is unit.

In our seismic simulation, the thickness of the single soil layer was selected to be 100 m. Soils located at half depth, $H/2$, are assumed as a representative of the layer to determine the confining pressure σ . Equations 3 and 4 are used to determine the frequency-dependent shear wave velocities and quality factors, Q ; whereas their

global averages over frequency are used as frequency-independent parameters. The shear wave velocity and quality factor, Q , are uniform throughout the frequency range for the frequency-independent model.

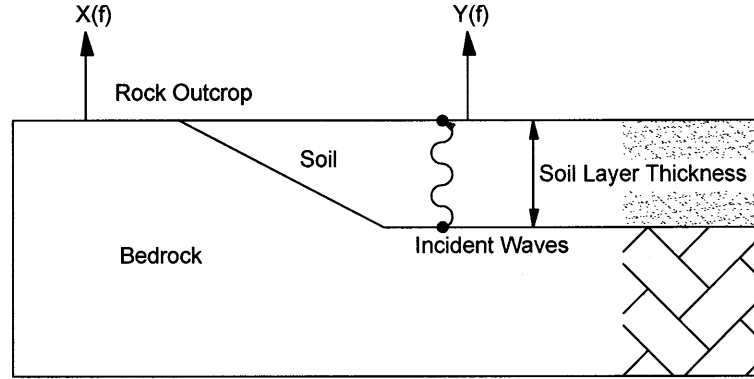


Figure 6. A Single Soil Layer over Bedrock Model Used in the Seismic Simulation

Simulation Results

Figure 6 shows the amplitude spectra by using the frequency-dependent and – independent parameters. The amplitudes of the two spectra both decrease with increasing of frequency. However, it appears that the difference in amplitude and resonant frequencies below 10 Hz is negligible; whereas it becomes more obvious with the increasing of frequency. The frequency-dependent spectra amplitude decreases much faster with the increase of frequency; and ratio of the difference reaches a maximum of 7 at 30 Hz. The Q at lower values between the frequency range of 10 and 30 Hz is likely a primary contributing factor in suppressing the frequency-dependent spectra amplitude. This confirms our conclusion on the effect of the aforementioned frequency-depended high-cut operator through kappa factor, beyond 10 Hz.

Conclusions

Frequency-dependent behavior, obtained from non-resonance method, in the seismic frequency range of 10^{-2} to 30 Hz of remolded soils, which mimic the surficial earth between depths of 20 and 60 m was reviewed. Each test consists of approximately 150 to 200 individual measurements per decade of frequency. To demonstrate the frequency-dependent effect on the amplification, a transfer function which was established for vertically propagating SH waves

passing through a single soil layer underlain by bedrock was used. The amplitude spectra of the transfer function from frequency-dependent and –independent simulations were compared. The results from the simulation indicate that the accurate characterization of frequency-dependent attenuation is instrumental in amplification spectra estimate in frequency range between 10 and 30 Hz; whereas constant Q assumption overestimates the amplification spectra at a maximum of 7 times at 30 Hz for SH waves traveling through a soil layer of 100 m thick. The effect of frequency-dependent attenuation does not appear to be considerable for frequencies less than 10 Hz, but should be considered where the site amplification is important beyond 10 Hz.

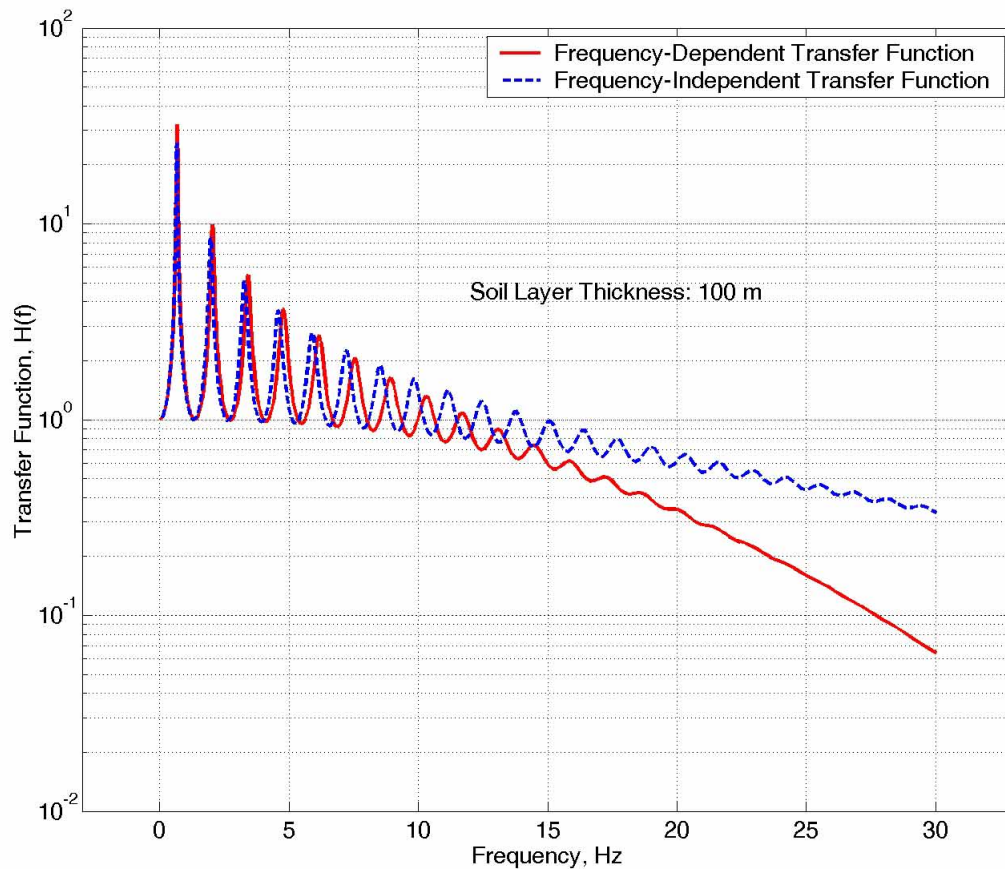


Figure 7. Comparison between the Frequency-Dependent and –Independent Seismic Simulation Results

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